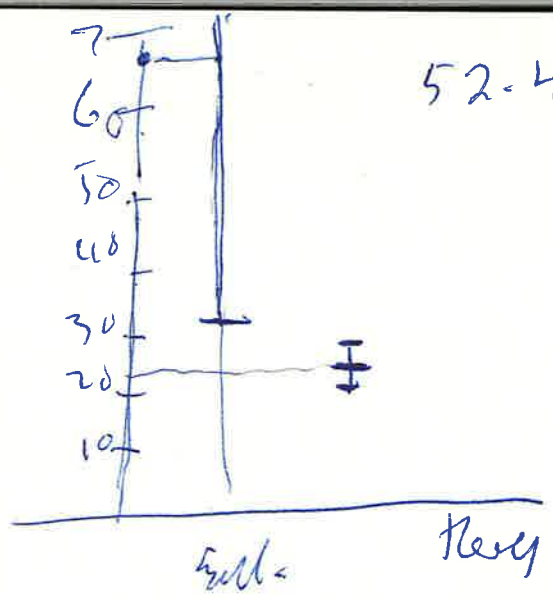


52.4

2.4 ± 0.6

24 ± 6

67 ± 35

$$\frac{500 \times 10^{-10}}{2 \times 2 \times 10^{4}} = \frac{10^{2} \times 10^{-10}}{10^{4}}$$

$$= 10^{-12}$$

$10^{-1} = 0.1$

$10^{-2} = 0.01$

$= 0.00\ldots1$

12

p.3. " This is just what a scientist means when he says T was an ad hoc explanation of e, namely T was devised for the express purpose of explaining e, so the explanation of e is guaranteed independent of whether T is true or false."
But is this the same as saying $\varepsilon = 1$?
Couldn't T be ad hoc in the sense of the quotation, but still $p(e | \sim T \& b) < 1$?

—

~~p3. "This is a situation in which~~
p3. "This is a situation in which Schaffner would claim that the theory was ad hoc..." I think a word of explanation as to why this is so might be in order (I had to puzzle over it for a moment).

THE LOGIC OF COMPARATIVE THEORY EVALUATION

by
M.L.G. Redhead

Chelsea College, London

, based on Zahar's account of novelty,

ABSTRACT

Schaffner's logic of comparative theory evaluation is criticised for an ~~incorrect~~ inappropriate ~~interpretation of Bayes's theorem.~~ analysis of ad hocness ~~A corrected~~ alternative analysis is given and extended to the case of multiple successful predictions by a theory. The application of the method to the appraisal of quantitative prediction is discussed ~~with examples. A Bayesian model is next presented for the general case in which successful and unsuccessful predictions are interspersed in the development of a research programme. An alternative approach incorporating the notion of methodological decision is discussed, and simple representations are provided for the life and death of a research programme.~~

[1967] account. He claims in a footnote[4] to have improved on Salmon's presentation. Unfortunately he so far "improves" the theorem that he actually renders it false.

Denoting by $P(T/b \& e)$ the probability of a theory T to be true in the light of background knowledge b and the positive outcome e of some experiment not part of b, by $P(T/b)$ the prior assessment of T, by $P(e/T \& b)$ the probability of obtaining e given T and b, and by $P(e/b)$ the probability of obtaining e on the basis of theories other than T" (Shaffner's italics), Schaffner writes

$$P(T/b \& e) = \frac{P(T/b) \cdot P(e/T \& b)}{P(e/b)} \qquad (1)$$

In a footnote[5] he explains $P(e/b)$ can be expanded in terms of a set of n "alternative competing theories" (my italics) in the form

$$P(e/b) = \sum_{i=1}^{n} P(T_i/b) \cdot P(e/T_i \& b) \qquad (2)$$

Now (1) is of course true only if $P(e/b)$ includes a contribution from T itself, which Schaffner explicitely denies in his definition of $P(e/b)$. This unfortunate oversight now makes nonsense of Schaffner's subsequent analysis. Because he assumes $P(e/b)$ is independent of $P(T/b)$ he makes his criterion for a theory to be ad hoc$_2$[6], depend essentially on the smallness of $P(T/b)$. The argument for $P(T/b)$ being small is essentially that it has no "theoretical support". This looks suspiciously like a reference to ad hoc$_3$, and Schaffner now appears to be claiming that a theory is ad hoc$_2$ in virtue of its being ad hoc$_3$. So he is not really giving an independent analysis of ad hoc$_2$ at all. But it is pointless to criticise his detailed arguments since they are all derived by using the wrong formulation of Bayes's theorem. The only statement which Schaffner gets right is where he discusses the "odds form" for comparing the evaluation of a theory with its rivals[7]. Since in taking the appropriate ratio $P(e/b)$ cancels out this is the one point at which his argument is not invalidated by his incorrect analysis of $P(e/b)$.

2. The Bayesian Analysis of Ad hocness

It is in fact a fairly simple matter to get an "internal" analysis of ad hoc$_2$ by using the correct form of Bayes's theorem, which we write as

$$P(T/b \& e) = \frac{P(T/b) \cdot P(e/T \& b)}{P(e/T \& b)P(T/b) + P(e/\sim T \& b)P(\sim T/b)} \qquad (3)$$

where $\sim T$ denotes the negation of T.

degree of belief

Note that we follow Schaffner in interpreting P(A/B) as the probability that A is true given that B is true. Note also that e is supposed in equation (3) to refer to a prediction made by the theory T (we could perhaps write e_T to emphasize this important point) so P(e/~T & b) means the probability that the prediction e_T derived from the theory T is a true prediction given that the background information b is true but the theory T is false (i.e., its consequences are not guaranteed to be true although "by accident" they may be true).

Writing $P(T/b) = x$, $P(e/\sim T \& b) = \varepsilon$

and taking $P(e/T \& b) = 1$ and using $P(\sim T/b) = 1 - x$

$$P(T/b \& e) = \frac{x}{x + \varepsilon(1 - x)} \qquad (4)$$

We define an enhancement ratio π by

$$\pi = \frac{P(T/b \& e)}{P(T/b)}$$

where using (4) we obtain the simple result

$$\pi = \frac{1}{x + \varepsilon(1 - x)} \qquad (5)$$

We can now explain that if a theory T is ad hoc$_2$ with respect to the experiment e then $\varepsilon = 1$, i.e. the explanation of e by T in no way depends on the truth or falsehood of T, both of which eventualities lead with certainty to the result e. This is just what a scientist means when he says T was an ad hoc explanation of e, namely T was devised for the express purpose of explaining e, so the explanation of e is guaranteed independent of whether T is true or false. To show the consistency of our analysis if we put $\varepsilon = 1$ in (5) we get $\pi = 1$, so the posterior and prior probabilities of T are equal (there is no enhancement) and this again is just what we expect from an ad hoc explanation of e, namely e itself gives us no additional information for assessing the truth of T.

In our analysis we notice that if $x \ll \varepsilon \ll 1$, then $\pi \simeq 1/\varepsilon$, so in this case we get a big enhancement and the theory is far from being ad hoc. This is a situation in which Schaffner would claim that the theory was ad hoc, which highlights the way in which his analysis differs from ours.

Notice that under these conditions $P(e/T) \geq \varepsilon$, we see that

2. The Case of Multiple Predictions

To develop our analysis a little further we can consider how a theory builds up a favourable appraisal as it makes a number of successful predictions e_1, $e_2 \ldots e_n$ say. Denote by P_s the posterior probability $P(T/b\,\&\,e_1\,\&\,e_2 \ldots \&\,e_s)$ after s successful predictions. Assuming the predictions are quite independent and for simplicity are all associated with the same value of ε, we can clearly write

$$P_n = \pi^{(n)} \times \pi^{(n-1)} \times \cdots \times \pi^{(1)} \cdot P_o$$

where $P_o = x$ according to our previous notation

and

$$\pi^{(s)} = \frac{1}{P_{s-1}(1-\varepsilon) + \varepsilon}$$

$$P_{s+1} = \pi^{(s+1)} \cdot P_s$$

The solution of this recursion is by inspection or more simply by replacing ε by ε^n in the formula for P_1 (see (4) above). We obtain

$$P_n = \frac{1}{1 - \varepsilon^n + \varepsilon^n/x} \qquad (6)$$

We can also ask what is the probability for the $(n+1)^{th}$ prediction e_{n+1} being correct if the theory has already made n successful predictions $e_1 \ldots e_n$. Denoting this probability by $P(e_{n+1})$ we clearly obtain the result

$$P(e_{n+1}) = \varepsilon + \frac{1-\varepsilon}{1 - \varepsilon^n + \varepsilon^n/x} \qquad (7)$$

If ε is a small quantity (i.e. $\ll 1$) which will be the case if T is non-ad hoc$_2$ with respect to all the predictions, we can write the following formulae which will be perfectly satisfactory for the subsequent discussion

$$P_n \simeq \frac{1}{1 + \varepsilon^n/x} \qquad (8)$$

$$P(e_{n+1}) \simeq \varepsilon + P_n \qquad (9)$$

$$\pi^{(n)} \simeq \frac{1}{\varepsilon + P_n - 1} \qquad (10)$$

P_{n-1}

We note the following features

(1) The value of Pn depends entirely on the ratio ε^n/x
For $\varepsilon^n/x \gg 1$, $P_n \ll 1$
and for $\varepsilon^n/x \ll 1$, $P_n \simeq 1$
At the critical point $\varepsilon^n = x$, we have Pn $\simeq 1/2$.
So if initially $x \ll \varepsilon$ as n increases Pn will rise steeply as we reach the value n = $\ell n\, x/\ell n\, \varepsilon$

(2) So long as Pn $\ll \varepsilon$, we have $P(\ell_{n+1}) \simeq \varepsilon$, but as Pn builds up towards unity, so does $P(\ell_{n+1})$.

(3) So long as $P_{n-1} \ll \varepsilon$, $R^{(n)} \simeq 1/\varepsilon$, but as P_{n-1} builds up towards unity the enhancement factor $R^{(n)}$ also tends to unity.

the accompanying figure

To take a concrete case we illustrate in ~~Fig.1~~ P_n and $P(\ell_{n+1})$ as functions of n for the particular choice x = 0.0 1, ε = 0.1.

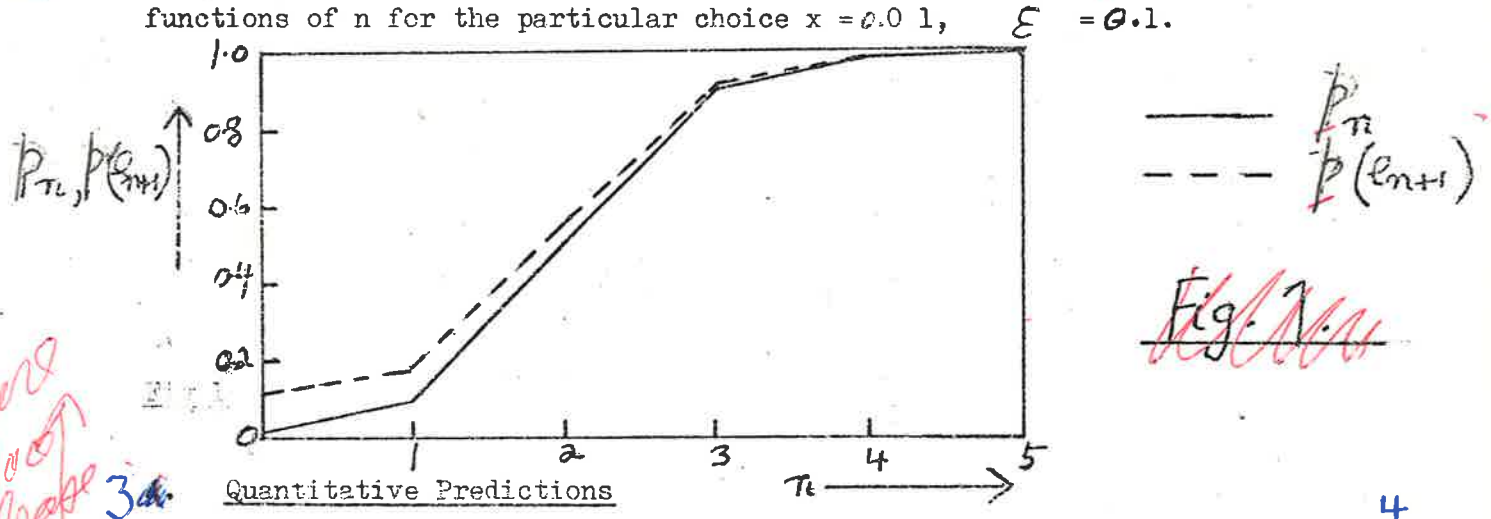


Fig. 1

3a. Quantitative Predictions

4

We can apply this analysis to the important case of quantitative predictions. Suppose a theory T predicts correctly an experimental result which is known to an accuracy of n significant figures. We assume as part of our background knowledge that the order of magnitude of the result is known, i.e we disregard the prediction of zeros occurring before or after the significant figures in the experimental result. As a concrete illustration we cite the theoretical predictions of quantum electrodynamics of the anomaly in the hydrogen spectrum known as the Lamb shift and the anomaly in the magnetic moment of the electron. For example the latter is now known experimentally to be 0.001159656.7 ± 3.5 Bohr magnetons whereas the theoretical prediction is 0.001159652.4 ± 0.6 Bohr magnetons. 5 ~~(For a good account of the fluctuating agreement between theory and experiment for this phenomenon the reviews by Lautrup, Peterman and de Raphael in their [1972] or Rich and Wesley in their [1972] may be consulted.)~~ Clearly

We thus have remarkable agreement to six significant figures.

If we regard the prediction of each significant figure as an independent event then the appropriate value to take for ξ is 0.1 since a false theory would have ten equal possibilities for filling in each digit. The question of what value to take for x is somewhat arbitrary. In agreement with Schaffner we do not follow a purely logical approach and set x = 0. For our purposes x reflects the scientist's confidence in the new theory T. One could argue that a scientist would not spend great efforts developing the consequences of a theory he did not believe in. By analogy with the situation in the Bayesian analysis of significance testing (see for example Redhead [1974]) we could take $x = \frac{1}{2}$. Perhaps more realistically we should take x around 0.01 and adapt the sociological rule that unless a scientist has a one percent level of confidence in the truth of his theory he would not seriously investigate it. With this choice of parameters we see that the build-up of confidence in a theory which makes correct quantitative predictions, as the accuracy of the experiment increases, would be illustrated by the graph for Pn in Fig.1. Of course $P(e_{n+1})$ is only given by our analysis so long as other factors which could potentially influence the results are known not to be significant. For a certain value of n this condition will fail. For example in the case of the anomalous magnetic moment of the electron the effect of hadronic couplings introduce theoretical uncertainties which would ultimately make the prediction of the theory unreliable. On the other hand the Lamb shift has so far been measured with an accuracy less by a factor of twenty, so the physicist's confidence in refined quantitative predictions here is very high.

5. A Bayesian Model for the Methodology of Research Programmes

We now want to extend our analysis to the case where the theory T consists of two parts $T = C \& B$ where C is the hard core and B an auxiliary hypothesis selected from a set $\{B_i\}$ which represents the possible articulations of the protective belt in Lakatos's terminology (see for example his [1970]).

We write

$$P(C/e \& b) = \frac{P(e/C \& b) \cdot P(C/b)}{P(e/C \& b) \cdot P(C/b) + P(e/\sim C \& b) \cdot P(\sim C/b)}$$

and expand

$$P(e/C \& b) = P(e/C \& B \& b) \cdot P(B/C \& b) + P(e/C \& \sim B \& b) \cdot P(\sim B/C \& b)$$

Introducing abbreviations $P(C/b) = x$, $P(B/C \& b) = y$, $P(e/C \& B \& b) = k_1$, $P(e/\sim C \& b) = k_2$, and setting $P(e/C \& B \& b) = 1$, $P(\sim B/C \& b) = 1-y$, $P(\sim C/b) = 1-x$, then the enhancement ratio for the hard core is Π_C where

$$\Pi_C = \frac{P(C/e \& b)}{x} = \frac{y + k_1(1-y)}{(y + k_1(1-y))x + k_2(1-x)}$$

If we take $k_1 = k_2 = k$ say we get

(11) $$\Pi_c = \frac{y + k(1-y)}{xy + k(1-xy)}$$

Similarly Π_B, the enhancement ratio for the auxiliary hypothesis, is of the form

(12) $$\Pi_B = \frac{x + k(1-x)}{xy + k(1-xy)}$$

if we assume $P(e/\sim c \,\&\, B \,\&\, b) = k$ and $P(e/\sim B \,\&\, b) = k$ for simplicity. For an ad hoc prediction we take $k = 1$ to get $\Pi_c = \Pi_B = 1$ so again there is no enhancement in this case. For a non-ad hoc prediction take $k \ll xy$ Then $\Pi_c \simeq 1/x$

Similarly $\Pi_B \simeq 1/y$, so we can now get big enhancements leading in our approximation limit to $P(c/b \,\&\, e) = 1$ and $P(B/b \,\&\, e) = 1$ after confirmation. This is true irrespective of the magnitude of x and y.

We turn now to the interesting case of refutation and write

$$P(c/b \,\&\, \sim e) = \frac{P(\sim e/c \,\&\, b) \cdot P(c/b)}{P(\sim e/c \,\&\, b)P(c/b) + P(\sim e/\sim c \,\&\, b) \cdot P(\sim c/b)}$$

where

$$P(\sim e/c \,\&\, b) = P(\sim e/c \,\&\, B \,\&\, b) \cdot P(B/c \,\&\, b) + P(\sim e/c \,\&\, \sim B \,\&\, b) \cdot P(\sim B/c \,\&\, b)$$

We now substitute

$P(\sim e/c \,\&\, B \,\&\, b) = 0$

$P(\sim e/\sim c \,\&\, b) = 1 - k_2$, $P(\sim e/c \,\&\, \sim B \,\&\, b) = 1 - k_1$, to give in this case, again assuming $k_1 = k_2 = k$

(13) $$\Pi_c = \frac{1-y}{1-xy}$$

and under the same assumptions as we used in deriving (12)

(14) $$\Pi_B = \frac{1-x}{1-xy}$$

These formulae for Π_c and Π_B are independent of the value of k.

We note at once that for refutation Π_c and Π_B are always less than unity, as we would expect, but suppose we have x near to 1 and y small, then for $x = 1$, $y = 0$ we get $\Pi_c = 1$ so there is no loss of confidence in this limit. But in the same limit $\Pi_B = 0$, so refutation acts asymmetrically on C and B provided we take x large and y small as reflecting the methodology of research programmes.[10]

Notice in particular we can explicate asymmetric refutation without having to assume k small.

The life and death of a research programme can now be modelled using our two formulae (11) and (13) to allow for the effects of successful and unsuccessful predictions. In Fig. 2 we illustrate the following sequence - two alternating successes and failures followed by a run of failures, where we have assumed as parameter values x = 0.9, y = 0.5 and k = 0.1. The probability P_C for the hard core and the probability for the current articulation of the protective belt P_B are both shown. After each refutation we take $P_B = 0.5$ again for the new auxiliary hypotheses introduced to account for the anomaly. We are assuming here the simple situation in which each articulation of the protective belt is triggered by an anomaly[11].

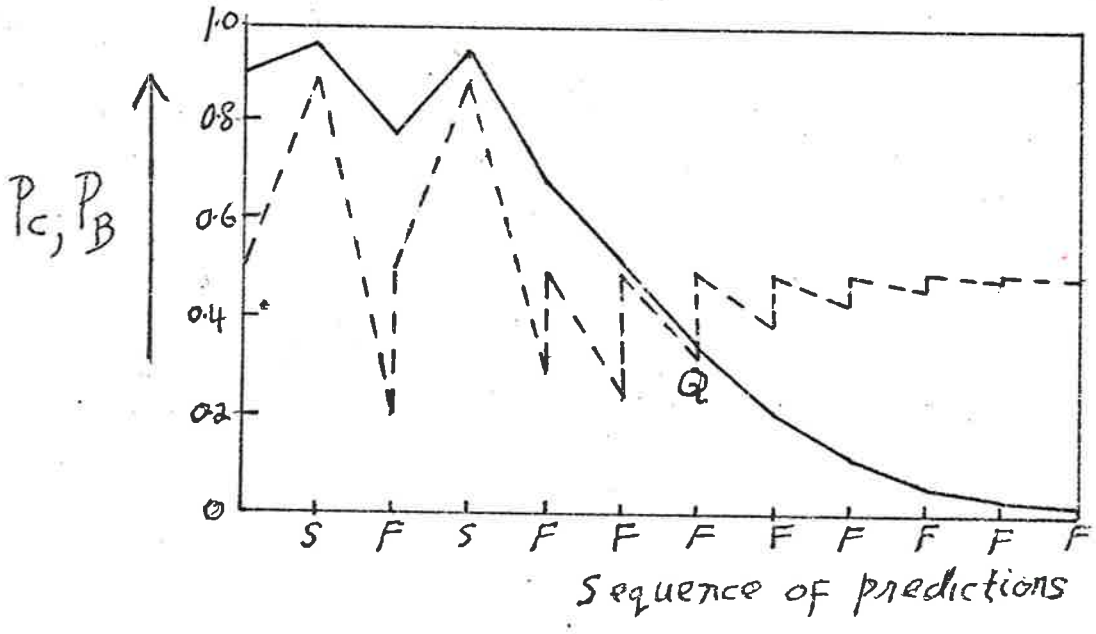


Fig. 2

——— P_C
------ P_B

S = Successful prediction
F = Failed prediction.

Suppose after the fourth failure in the degeneration sequence where P_C has fallen to 0.21 we had got a success, P_C would then recover to the value 0.60. Even with P_C down to .06 at the sixth failure in the degeneration sequence a success would lift P_C to 0.26.

At the "crossover" point of the P_C and P_B curves (marked Q in Fig. 2) it would become rational to change the hard core rather than the protective belt, i.e. we now have an internal rational criterion for abandoning a research programme, independent of any competing programme.

At first sight we might suppose that if we have a run of successful predictions followed by a failure then the asymmetry in the effect of failure on P_C and P_B which was large initially would be eliminated as P_C and P_B were both now increased to values very close to unity. We shall now show that this rather natural assumption is unwarranted and that in our model the asymmetric effect of failure on P_C and P_B is preserved however many successful predictions precede the failure. What we shall prove is the following:

After n successful predictions let P_C increase from $P_C(0)$ to $P_C(n)$ and P_B from $P_B(0)$ to $P_B(n)$.

Let the effect of a refutation on P_C and P_B after n successful predictions be denoted by $\Pi_C(n)$ and $\Pi_B(n)$ respectively.

Then we prove

$$\frac{\Pi_B(n)}{\Pi_C(n)} = \frac{\Pi_B(0)}{\Pi_C(0)} = \frac{1-P_C(0)}{1-P_B(0)}$$

a result which is independent of n

We have at once the result

$$\frac{\Pi_B(n)}{\Pi_C(n)} = \frac{1-P_C(n)}{1-P_B(n)} \quad \text{(see equations (13) and (14))}.$$

Now use equation (14) to obtain the result

$$P_C(n+1) = \frac{P_B(n) + R(1-P_B(n))}{P_C(n)P_B(n) + R(1-P_C(n)\cdot P_B(n))} \cdot P_C(n)$$

So $1 - P_C(n+1)$

$$= \frac{R(1-P_C(n))}{P_C(n)\cdot P_B(n) + R(1-P_C(n)\cdot P_B(n))}$$

Similarly we can show

$$1 - P_B(n+1) = \frac{\cdot R(1-P_B(n))}{P_C(n)\cdot P_B(n) + R(1-P_C(n)\cdot P_B(n))}$$

$$\therefore \frac{1-P_C(n+1)}{1-P_B(n+1)} = \frac{1-P_C(n)}{1-P_B(n)}$$

which applied recursively yields the result

$$\frac{1-P_C(n)}{1-P_B(n)} = \frac{1-P_C(0)}{1-P_B(0)} = \frac{1-x}{1-y}$$ unworked notation,

which is independent of n.

Hence we get for refutation after n successes

$$\frac{\Pi_B(n)}{\Pi_C(n)} = \frac{1-x}{1-y}$$

In our own example $x = 0.9$, $y = 0.5$, so $\frac{\Pi_B(n)}{\Pi_C(n)} = 1/5$

and the effect of refutation on B is five times as high as on C for all values of n, although of course it is also true that $P_B(n)$ and $P_C(n)$ both tend rapidly towards unity with increasing n.

We stress that in this model the asymmetric effect of refutation on the protective belt and the hard core is directly accounted for at all stages by the initial value of P_C being much higher (closer to unity) than we assume for P_B.

6. An Alternative Approach - The Methodological Decision

In the model we have just discussed the effect of refuation on P_C is given a rational (Bayesian) basis. But other approaches to the methodology of theory evaluations can best be understood as a mixture of rational (Bayesian) and methodological (extra-Bayesian) decisions. We shall now illustrate some of these ideas.

Thus faced with an anomaly there are various methodological prescriptions which tell the scientist how to proceed. For example there is a crude Popperian picture which tell him to reject a theory if it is refuted. But the tenacity of theories is reflected in Lakatos's methodology of research programmes.

To analyse this situation we again write T as a conjunction of a hard core C and an auxiliary hypothesis B_i, which belongs to a set $\{B_i\}$ of adjustable hypotheses in the protective belt, adjustable that is in accordance with the positive heuristic of the research programme. Thus $T = C \& B_i$.
Now suppose the theory T makes a false prediction e_1. Since this is impossible if the theory T were true we must write $P(e_1/T\,\&\,b) = 0$ in equation (3) which leads immediately to $P(T/b \& e_1) = 0$. If we assume C and B_1 are independent hypotheses for the sake of simplicity, then we can write

$$P(T/b \& e_1) = P(C/b \& e_1) \times P(B_1/b \& e_1) = 0$$

So we are led to the condition
$P(C/\mathcal{L} \& e_1) = 0$ or $P(B_1/\mathcal{L} \& e_1) = 0$. Which alternative we take cannot be decided by the probability calculus alone with the information presumed so far alone at our disposal. It is a matter for methodological decision. Thus the Lakatos methodology can be expressed very simply. We choose to take $P(B_1/\mathcal{L} \& e_1) = 0$ and leave $P(C/\mathcal{L} \& e_1)$ unchanged in the face of a false prediction e_1. We now invoke a new hypothesis B_2 which enables $C \& B_2$ to explain e_1. This is an ad hoc manoeuvre for $C \& B_2$ with respect to e_1 so $P(C \& B_2/\mathcal{L} \& e_1)$ can only be enhanced by new successful novel predictions. Suppose e_2 is successfully predicted by $C \& B_2$. Then our previous analysis gives

(15) $$P(C \& B_2/\mathcal{L} \& e_1 \& e_2) = \Pi \times P(C \& B_2/\mathcal{L} \& e_1)$$

where Π is a suitable enhancement factor. We can rewrite (15) as

(16) $$P(C/\mathcal{L} \& e_1 \& e_2) \times P(B_2/\mathcal{L} \& e_1 \& e_2) = \Pi \times P(C/\mathcal{L} \& e_1) \times P(B_2/\mathcal{L} \& e_1)$$

But what we are actually interested in is how the credibility of the hard core behaves as the research programme progresses. To consider this question we must "distribute" the enhancement ratio between the hard core and the protective belt. But again the probability calculus alone cannot answer this question. Again we have to make a methodological decision. But this decision must be constrained so as not to transgress the rules of the probability calculus. The situation is rather like Popper's formal account of concepts like verisimilitude. We seek a simple formula that satisfies certain conditions. We formulate these conditions for our problem as follows.

In a suitably abbreviated notation we let A and B stand for two probabilities and we write

$$\Pi_{.A.B} = (\Pi_{A \cdot A}) \times (\Pi_{B \cdot B})$$

so the enhancement factors Π is divided between A and B according to the factors Π_A and Π_B which will be functions of Π, as well as each depending on A and B. Regarding A and B as functions of Π we list the following restrictions, assuming $A \not\equiv B$ without loss of generality.

(1) $\Pi_A(1) = \Pi_B(1) = 1$

(2) $\Pi_A\left(\frac{1}{AB}\right) = \frac{1}{A}$, $\Pi_B\left(\frac{1}{AB}\right) = \frac{1}{B}$

(3) $\Pi_A(\Pi)$ and $\Pi_B(\Pi)$ are monotonically increasing functions of Π in the range $(1, 1/AB)$.

(4) $\Pi_A \cdot A \geq \Pi_B \cdot B$ for all Π in the range (1,1/AB)

(5) $\Pi_A \cdot \Pi_B = \Pi$

(6) Writing $\Pi_A(\Pi) = F(A, B, \Pi)$

$$F(A, B, \Pi'\Pi'') = F(A, B, \Pi') \times F\left[F(A,B,\Pi') \cdot A, \frac{\Pi}{F(A,B,\Pi')} \cdot B, \Pi''\right]$$

with a similar condition on $\Pi_B(\Pi)$

Condition (2) arises from noting that the maximum value of Π is 1/AB since we require that $\Pi \cdot AB$ is a probability, the maximum value for Π_A is 1/A since $\Pi_A \cdot A$ is to be a probability, and similarly for Π_B.

Condition (4) means that enhancement never leads to a "cross-over" in the assessment of A and B.

Condition (6) means that our rule for arriving at A can be built up consistently in stages.

Conditions (1), (3) and (5) require no comment. It is perhaps surprising that a simple formula for Π_A can be obtained satisfying all these conditions, viz.

(17) $$\Pi_A = \frac{\Pi(1-AB)}{1-A+A(1-B)\Pi}$$

Π_B is then given trivially as Π / Π_A.

(18) $$\Pi_B = \frac{1-A+A(1-B)\Pi}{1-AB}$$

For $A < B$ we interchange the formulae for Π_A and Π_B, at the same time exchanging A and B in these formulae. It is conjectured that (17) and (18) is the only simple solution satisfying the conditions (1) to (6).

The only other property we might like to impose is

(7) $\Pi_A = \Pi_B = \sqrt{\Pi}$ for $A = B$

Our solution (17) and (18) does not satisfy condition (7). There appears to be no simple solution that satisfies all of conditions (1) to (7) [12]. Since we are only interested in a model which will bring out the general features of different methodologies we shall adopt our solutions (17) and (18).

Another feature we might want to build into our model is a memory[13]. That is to say after a number m of unsuccessful predictions we do not choose to leave the probability of the hard core entirely unchanged but "depress" it at the m^{th} refutation by a memory factor F(m) which begins to cut off at some critical value m_0.

We can now put all these features together to obtain a model for the following very general problem.

Suppose a research programme makes n_m successful predictions following the $(m-1)^{\underline{th}}$ false prediction by suitable articulation of the positive heuristics.

We denote by $P_C(n_1, n_2 \ldots n_m)$ the posterior probability for the hard core C after this sequence of successes and failures. Then we can write the following recursion for Pc

$$P_C(n_1, n_2 \cdots n_m) = P_C'(n_1 \cdots n_{m-1}) \times \textcircled{H}\left(\sqrt{x},\ P_C'(n_1 \cdots n_{m-1}),\ \Pi^{(n_m)}\right)$$

where $P_C'(n_1 \cdots n_{m-1}) = F(m-1) \times P_C(n_1 \cdots n_{m-1})$

and $$\Pi^{(n_m)} = \Phi\left(\varepsilon,\ n_m,\ \sqrt{x}.\ P_C'(n_1 \cdots n_{m-1})\right)$$

$$\Pi^{(n_1)} = \Phi\left(\varepsilon,\ n_1,\ x\right)$$

with $P_C(n_1) = \sqrt{x}\ \textcircled{H}\left(\sqrt{x}, \sqrt{x}, \Pi^{(n_1)}\right)$

and $$\textcircled{H}(A, B, C) = \frac{C(1-AB)}{1-B+B(1-A)C}$$

$$\Phi(A, B, C) = \frac{1}{C(1-A^B)+A^B}$$

$$F(m) = \frac{1}{1+\exp(m-m_0)}$$

In these formulae $$x = P(c \not\vdash B_i / b),\quad \varepsilon = P(e/\wedge(c \not\vdash B_i) \not\vdash b)$$

We have assumed that initially equal probabilities $\sqrt{x}$ are ascribed to C and B_1 and that each successive articulation of the protective belt starts again with probability $\sqrt{x}$.

We have chosen a simple form for F(m) with suitable cut off properties.

Taking the parameter values $m_0 = 3$, $\sqrt{x} = 0.01$, $\varepsilon = 0.1$ and assuming for purposes of illustration the case where successful and unsuccessful predictions alternate, the account derived from our model is shown in Fig.3, where graphs for pure Lakatos (no memory factor) and pure Popper (refutation takes us back to square one, i.e Pc falls to $\sqrt{x}$ after each refutation') are also given for comparison.

Certainly the memory models look the most realistic although we have chosen a very simple type of memory - a more sophisticated version would take due account of small α_n vanishing values of n_m as m increases, corresponding to the informal notion of degeneracy in the research programme.

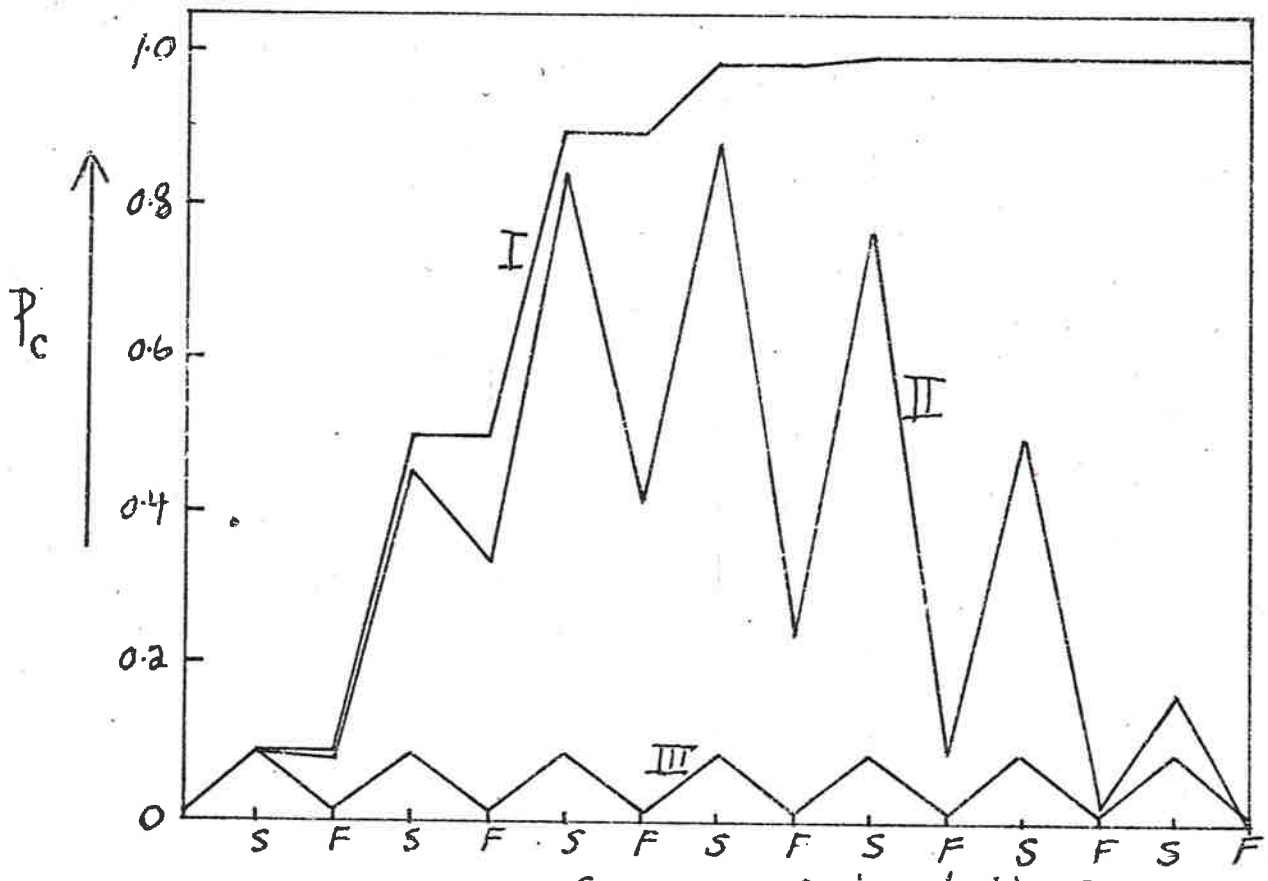


Fig.3

I Pure Lakatos : refutation leaves Pc unchanged.

II Pure Lakatos plus memory: refutation cuts Pc by a memory factor F(m).

III Pure Popper : refutation takes Pc back to $\sqrt{x}$.

Let us very briefly compare our new model with the previous one presented in Section 5. In the first place the new model assumes that B and C are independent

after refutation - this is explicitely denied in the purely Bayesian model. Of course the first model supposes more information about more complicated conditional probabilities such as $P(e/c\&\sim b\&b)$ etc, whereas the second model assumes we know only the value of $P(e/\sim(c\&b)\ \&\ b\ )$ as our input information. Naturally our methodological decisions in the second model could be re-expressed as conditions on the extra conditional probability assignments assumed in the first model. But the status of these assignments is now quite different. For the first model we derived the effect of refutation from these assignments. In the second model we effectively constrain these assignments by the decisions we choose to make about the effects of refutation. Finally we note that in the second model we can explain the progress of the research programme even allowing P_c and P_b initially to have the same low value, whereas the success of the first model depends critically on taking these as unequal. At all events we believe that we have successfully distinguished the possible roles of Bayesian or semi-Bayesian models in accounting for the life and death of a research programme.

FOOTNOTES

1 Zahar [1973] p.101.

2 Zahar [1973] p.103.

~~3 Schaffner [1974] p.69-73.~~ ← Insert Footnote 3 here

~~4 Ibid. Footnote 3 p.70.~~

~~5 Ibid. Footnote 2 p.70.~~

6 Schaffner actually ~~works in terms of hypotheses being ad hoc but we want to keep the argument as simple as possible.~~

~~7 Ibid. p.72.~~

~~4~~ 8. According to Rich and Wesley [1972] the known Radionic contribution to the electron anomaly would affect the tenth significant figure.

~~9 This method of analysing Bayes's theorem is due to Jon Dorling.~~

10 The asymmetric effect of refutation in the Bayesian model was pointed out to me by Jon Dorling who was responsible for the ideas behind the preceding analysis.

11 Lakatos of course stresses in his [1970] that realistic research programmes can progress in an ocean of anomalies which are ignored, the articulation of the protective belt being carried along by the internal momentum of the positive heuristic of the programme. Our 'rational reconstruction' of Lakatos's methodology, like all rational reconstructions, is a caricature.

12 I am grateful to Jon Dorling for obtaining the following rather complicated solution for conditions 1 to 7, viz.

$$\Omega_A = \frac{B + R(1-B)}{AB + R(1-AB)}$$

where R is the unique root in the range $0 \leq R \leq 1$ of the following quadratic equation

$$R^2\{(1-A)(1-B) - (1-AB)^2\Omega\} + R\{A+B-2AB - 2AB(1-AB)\Omega\} + AB(1-AB\Omega) = 0$$

We leave the derivation of this result as a challenge to the interested reader.

13 ~~I am grateful to Noretta Koertge for suggesting this.~~

7. Ω_A is easy to see that ~~it~~ our confidence in a theory at a given level of accuracy for experiment between theory and experiment does not depend on the particular scale of notation used to express the result.

Footnote B

Shimony's concept of commitment to a theory (see his [1970] pp. 94-95) is what we have in mind. ~~Scientists believe~~ The degree to scientist's ~~the~~ belief that ~~the~~ degree of commitment measures ~~the~~ a theory T belongs to the equivalence ~~class~~ class of all theories which give the same true operational predictions within the ~~context~~ domain of current experimentation. and that the "true" theory ~~explains~~ generalizes" in some sense the concepts embodied in T. Commitment measures our belief, not that a theory is true, but that it points the way to the truth.

Footnote 3 What we have shown is that $\varepsilon = 1$ is a necessary condition for T to be an ad hoc$_2$ explanation of e. To justify $\varepsilon = 1$ as a sufficient condition we must invoke a principle of insufficient reason, viz. if there is no reason for the community of scientists to entertain work non-vanishing prior probability only theories which explain e, then they will not do constrain their choice of alternative theories.

~~Footnote 4 See Hylleraas [1963] p. 427~~

Footnote 5 See values quoted in Calmet et al. [1977]. For a good account of the fluctuating agreement between theory and experiment the reviews by Lautrup, Peterman and de Rafael [1972] or Rich and Wesley [1972] may be consulted.

Footnote 4

The successful detailed quantitative predictions of a theory in respect of phenomena quite different from those which the theory was originally proposed to deal with lend the theory added weight and attract the attention of scientists. To take an example at random, I allude to the reference to his early work on the ~~use~~ of Helium Hydrides contained in his [1963] (p. 42) "The end result of my calculations/was ~~calculated~~ admired --- quality of the validity of wave mechanics -- in the strict numerical sense".

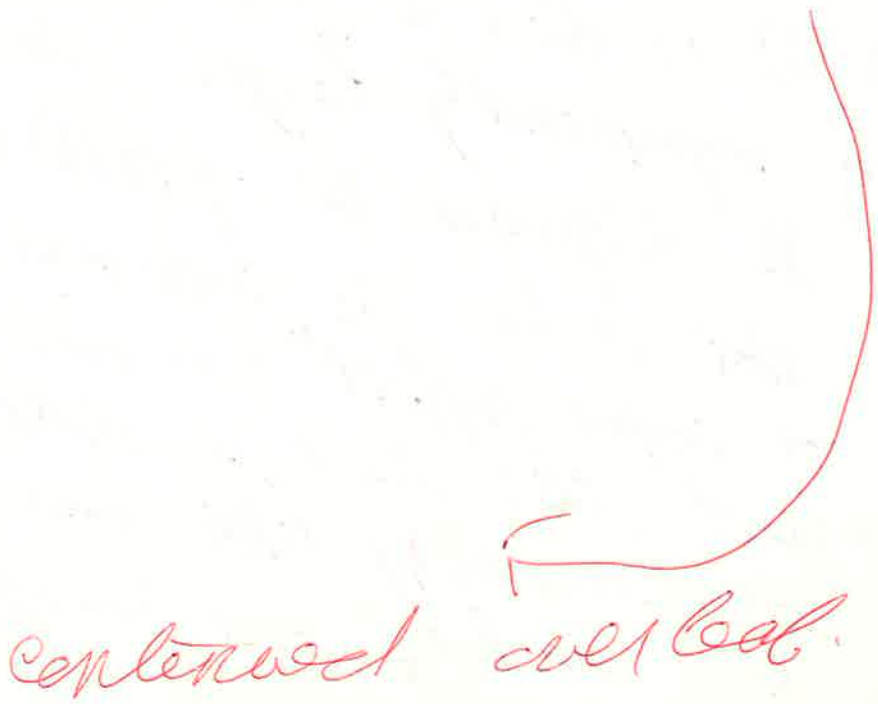
The logic of Empirical theory evaluation

Insertions

Insertion A

% T explains e we can set $P(e/T\&a)=1$
in this case
~~to get~~ ~~$P(T/a\&e)$~~ ~~$= \lambda$~~ we ~~can state~~ claim

$$P(T/a\&e) = P(T/a)/P(e) \quad (2)$$



continued overleaf

~~Insertions~~ — ~~The Logic of Corroborant Theory Evaluation~~

Schaffner proceeds to discuss ad hoc-ness as a property of an hypothesis or a ~~constituent~~ constituent part of a theory, but in order to keep the argument as simple as possible we shall follow Zahar in considering the ad hoc-ness of theories. ~~Translated into these terms~~ Schaffner's idea is that ~~an ad hoc~~ ~~theory, is devised to account for which~~ ~~a~~ a theory T gives an ad hoc explanation of an experimental result e if $p(T/e)$ is close to zero and $p(e/e)$ is significantly larger than $p(T/e)$. The argument for $p(T/e)$ being small is that it has no ~~empirical~~ "theoretical ~~support~~ empirical support other than e itself". (or indeed this looks suspiciously like ~~a~~ reference to ad hoc$_3$, and Schaffner

now appears to be clearer that a theory is ad hoc$_2$ partly in virtue of its being ad hoc$_3$. So to is not really giving an independent analysis of ad hoc$_2$ at all.

Schaffner explicates ~~novelty~~ the lack of novelty in the production of e by a high value ~~of~~ historical ~~temporal~~ of $p(e/h)$. This is an sense of novelty, which clearly differs from Zahar's usage

Rider Ward

the lack of novelty in the production of e is associated by Schaffner with a high value for $p(e/h)$. On Zahar's account this is a necessary condition for lack of novelty, but not a sufficient condition. We proceed to show how essay Zahar's notion of novelty one can get an "internal" Bayesian analysis of ad hoc$_2$

Rider Ward

We express Bayes's theorem in the following familiar way

However Schaffner ~~has some objections about Zahar~~ says that "Zahar's ~~account~~ analysis is weakened by his account of novelty", so

---

A' Notice that $p(e/h) = x + \varepsilon(1-x)$ is equal to unity if $\varepsilon = 1$, but that $p(e/h) = 1$ is ~~can be~~ achieved for $x = 1$ whatever the value of ε, so the explication of novelty in terms of the law $p(e/h)$ ~~and~~ ~~small~~ (Schaffner) and small ε (Zahar) are by no means equivalent.

B

Effectively Schaffner requires R_x to be small as his condition of adhocness. He is thus concerned with the absolute value of the probability of a theory after it has explained some experimental finding. If this absolute value is still small the theory is to be regarded as an ad hoc explanation of the experiment. In our account the important aspect in assessing ad hocness is not the absolute value of the probability but the enhancement rates. Our point against Schaffner is not that his analysis may not explicate some legitimate sense of the ambiguous appellation ad hoc, but that it fails to explicate Zahar's very important notion of ad hoc$_2$. There is no inconsistency for Schaffner's part, since he finds Zahar's account of ad hoc$_2$ inadequate in respect of the definition & novelty involved, but we would maintain that Zahar's sense of ad hoc$_2$ is the one that ought to be explicated, since it is the one most importantly used in science.

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* Viz. with its temporal and historical associations,

4. Conclusion

The concept of a novel prediction plays a very important part in the way scientists assess theories confederate in theories. Of course on a purely instrumentalist ~~theories~~ use of theories the question of novelty is of no consequence ~~at all~~. But to a realist it is only in virtue of novel successful predictions that ~~the~~ progress of a theory towards the truth can be given one's belief in ~~the truth of~~ a theory can find rational justification. Our Bayesian analysis of comparative logic can be viewed ~~from the Bayesian point~~ if degrees of commitment are governed by Bayesian rationality constraints.